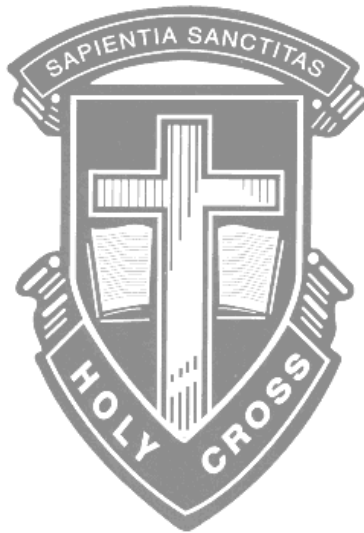


AP Calculus 2 Summer Review Packet



This review packet is to be completed by all students enrolled in AP Calculus 2. This packet must be submitted on the Monday of the first full week of class. It will be used as the First Assessment for the AP Calculus 2 Course

Name _____

AP Calculus 2

Evaluate each of the following limits.

1. $\lim_{x \rightarrow 5} (x^2 - 5x - 11)$

2. $\lim_{x \rightarrow 5} \frac{x+3}{x^2-15}$

3. $\lim_{x \rightarrow 0} \pi^2$

4. $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$

5. $\lim_{x \rightarrow \infty} \frac{10x^2 + 25x + 1}{x^4 - 8}$

6. $\lim_{x \rightarrow \infty} \frac{x^4 - 8}{10x^2 + 25x + 1}$

7. $\lim_{x \rightarrow \infty} \frac{\sqrt{5x^4 + 2x}}{x^2}$

8. $\lim_{x \rightarrow 6^+} \frac{x+2}{x^2 - 4x - 12}$

9. $\lim_{x \rightarrow 6^-} \frac{x+2}{x^2 - 4x - 12}$

10. $\lim_{x \rightarrow 6} \frac{x+2}{x^2 - 4x - 12}$

11. $\lim_{x \rightarrow 0^+} \frac{x}{|x|}$

12. $\lim_{x \rightarrow 0^-} \frac{x}{|x|}$

13. $\lim_{x \rightarrow 7^+} \frac{x}{x^2 - 49}$

14. $\lim_{x \rightarrow 7} \frac{x}{x^2 - 49}$

Let $f(x) = \begin{cases} x^2 - 5, & x \leq 3 \\ x + 2, & x > 3 \end{cases}$, Find...

15. $\lim_{x \rightarrow 3^-} f(x)$

16. $\lim_{x \rightarrow 3^+} f(x)$

17. $\lim_{x \rightarrow 3} f(x)$

18. $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

19. $\lim_{x \rightarrow \frac{\pi}{4}} 3 \cos x$

20. $\lim_{x \rightarrow 0} \frac{3x}{\cos x}$

21. $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 8x}$

22. $\lim_{x \rightarrow \infty} \sin x$

23. $\lim_{x \rightarrow \frac{\pi}{2}} \tan x$

24. $\lim_{x \rightarrow \infty} \sin \frac{1}{x}$

25. $\lim_{x \rightarrow 0} \frac{x^2 \sin x}{1 - \cos^2 x}$

26. $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$

27. $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$

28. $\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

29. $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

30. Is the function $f(x) = \begin{cases} x+7, & x < 2 \\ 9, & x = 2 \\ 3x+3, & x > 2 \end{cases}$ continuous? Explain.

31. Determine where the function $f(x) = \sec x$ is discontinuous.

32. Is the function $f(x) = \tan x$ continuous on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$?

33. For what value(s) of k is the function $f(x) = \begin{cases} 3x^2 - 11x - 4, & x \leq 4 \\ kx^2 - 2x - 1, & x > 4 \end{cases}$

continuous at $x = 4$?

34. For what value(s) of k is the function $f(x) = \begin{cases} -6x-12, & x < -3 \\ k^2 - 5k, & x = -3 \\ 6, & x > -3 \end{cases}$

Continuous at $x = -3$?

35. At what point is the removable discontinuity for the function $f(x) = \frac{x^2 - 5x - 24}{x^2 - x - 6}$?

36.

Find the derivative of each of the following functions.

$$37. f(x) = \frac{4x^3 - 3x^2}{5x^7 + 1}$$

$$38. f(x) = (x^2 - 4x + 3)(x + 1)$$

$$39. f(x) = (x + 1)^{10}$$

$$40. f(x) = 8\sqrt{x^4 - 4x^2}$$

$$41. f(x) = \left(\frac{x}{x^2 + 1} \right)^3$$

$$42. f(x) = \sqrt[4]{\frac{2x - 5}{5x + 2}}$$

$$43. y = \frac{4x^8 - \sqrt{x}}{8x^4}$$

$$44. y = \left(x + \frac{1}{x} \right) \left(x^2 - \frac{1}{x^2} \right)$$

$$45. y = \sqrt{\frac{x^2 + 1}{x^2 - 1}}$$

$$46. f(x) = \frac{x^2 - 3}{x - 3}$$

$$47. \text{ Find } \frac{dy}{dx} \text{ at } x = 1 \text{ if } y = \frac{t^2 + 2}{t^2 - 2} \text{ and } t = x^3$$

$$48. \text{ Find } \frac{du}{dv} \text{ at } v = 2 \text{ if } u = \sqrt{x^3 + x^2} \text{ and } x = \frac{1}{v}.$$

Find the derivative of each trigonometric function.

$$49. y = \sin^2 x$$

$$50. y = \cos x^2$$

$$51. y = \tan x \sec x$$

$$52. y = \sqrt{\sin 3x}$$

$$53. y = \frac{1 + \sin x}{1 - \sin x}$$

$$54. y = \sec \theta \tan 2\theta$$

$$55. y = \frac{\sec \theta}{1 + \tan \theta}$$

Use implicit differentiation to find the derivative of each equation.

$$56. \cos y - \sin x = \sin y - \cos x$$

$$57. x^3 - y^3 = y$$

$$58. x^2 - 16xy + y^2 = 1$$

$$59. x^2 + 4y^2 = 1$$

$$60. \text{ Find } \frac{dy}{dx} \text{ if } 16x^2 - 16xy + y^2 = 1 \text{ at } (1,1).$$

61. Find $\frac{dy}{dx}$ if $x^{1/2} + y^{1/2} = 2y^2$ at $(1,1)$.

62. Find $\frac{dy}{dx}$ if $x \sin y + y \sin x = \frac{\pi}{2\sqrt{2}}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

63. Find $\frac{d^2y}{dx^2}$ if $x^2 + 4y^2 = 1$

64. Find $\frac{d^2y}{dx^2}$ if $\sin x + 1 = \cos y$.

65. Find $\frac{d^2y}{dx^2}$ if $x^2 - 4x = 2y - 2$.

Find the derivative of each of the following functions.

66. $f(x) = \ln(x^4 + 8)$

67. $f(x) = \ln(3x\sqrt{3+x})$

68. $f(x) = x \ln(\cos 3x) - x^3$

69. $f(x) = e^{x \cos x}$

70. $f(x) = \log_{12}(x^3)$

71. $f(x) = \frac{\log_4 x}{e^{4x}}$

72. $y = \ln(e^x)$

73. $y = 10^{\log(x^2)}$

74. Find the equation of the tangent to the graph of $y = 3x^2 - x$ at $x = 1$.

Use the limit definition of derivative $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of each of the following functions.

75. $y = 3x^2 - x$

76. $f(x) = \sqrt{x+1}$

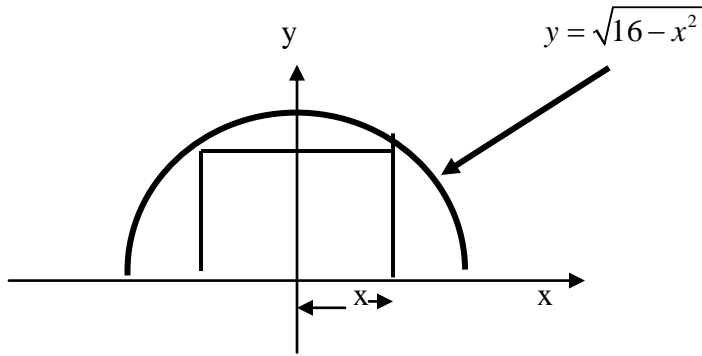
77. Find the equation of the tangent to the graph of $f(x) = \frac{1}{\sqrt{x^2 + 7}}$ at $x = 3$.

78. Find the coordinates where the tangent to the graph of $y = 8 - 3x - x^2$ is parallel to the x-axis.

79. Find the equation of the tangent and normal to the graph of $y = (x^2 + 4x + 4)^2$ at $x = 2$.

80. Max wants to make a box with no lid from a rectangular sheet of cardboard that is 18 inches by 24 inches. The box is to be made by cutting a square of side x from each corner and folding up the sides. Find the value of x that maximizes the volume.

81. A rectangle is to be inscribed in a semicircle of radius 4 with one side on the semicircle's diameter. What is the largest area this rectangle can have?



82. The range of a projectile is $R = \frac{v_o^2 \sin 2\theta}{g}$, where v_o is the initial velocity, g is the acceleration due to gravity and is a constant, and θ is the firing angle. Find the value of θ that maximizes the projectile's range.

83. If the position function of a particle is $x(t) = t^3 + 8t^2 - 2t, t > 0$, find the values of t where the particle is changing direction.

Find the linearization $L(x) = f(a) + f'(a)(x - a)$ at $x = a$ for each of the following functions at the given value of a .

84. $f(x) = x^3 - 2x + 3$ at $a = 2$.

85. $f(x) = \sqrt{x^2 + 9}$ at $a = -4$.

86. $f(x) = x + \frac{1}{x}$ at $a = 1$

87. $f(x) = \sqrt{x^2 + 9}$ at $a = -4$.

88. $f(x) = x + \frac{1}{x}$ at $a = 1$

89. $f(x) = \ln(x+1)$ at $a = 0$

90. Oil spilled from a tanker spreads in a circle whose circumference increases at a rate of 40 ft/sec. How fast is the area of the spill increasing when the circumference of the circle is 100π feet.

91. A spherical balloon is increasing at a rate of $27\pi \text{ in}^3/\text{sec}$. How fast is the radius of the balloon increasing when the radius is 3 inches?

92. Cars A and B leave a town at the same time. Car A heads due south at a rate of 80 km/hr and car B heads due west at a rate of 60 km/hr. How fast is the distance between the cars increasing after 3 hours?

93. A cylindrical tank with a radius of 6 meters is filling with fluid at a rate of $108\pi \text{ m}^3 / \text{sec}$. How fast is the height increasing?

94. The side of an equilateral triangle is increasing at a rate of 27 in/sec. How fast is the triangle's area increasing when the sides of the triangle are each 18 inches?

Evaluate each of the following integrals analytically.

95. $\int \frac{1}{x^4} dx$

96. $\int \frac{5}{\sqrt{x}} dx$

97. $\int \frac{x^3 + 7}{x^2} dx$

98. $\int (5x^4 - 3x^2 + 2x + 6) dx$

99. $\int (3x^{-3} - 2x^{-2} + x^4 + 16x^7) dx$

100. $\int (\cos x - 5 \sin x) dx$

101. $\int \sec x (\sec x + \tan x) dx$

102. $\int (\sec^2 x + x) dx$

103. $\int \frac{1}{(x+3)} dx$

104. $\int \frac{3}{5} e^x dx$

105. $\int e^{\ln(x^2-3x)} dx$

106. $\int \ln e^{\sqrt{x}} dx$

107. Use a Riemann sum to find the area under the curve $y = 2x - x^2$ from $x = 1$ to $x = 2$ using 4 subintervals.

108. Find the exact area under the curve $y = 2x - x^2$ from $x = 1$ to $x = 2$. Check using a graphing utility.

109. Find the area bounded by the function $f(x) = x^2 - 2$, the x-axis, over the closed interval $[0, 3]$. Check using a graphing utility. (Hint: Sketch the graph of the function over the given interval.)

Evaluate each of the following definite integrals *analytically*. Check your answer using a graphing utility.

$$110. \int_{-\pi/2}^{\pi/2} \cos x dx$$

$$111. \int_0^9 2x\sqrt{x} dx$$

$$112. \int_0^1 (x^4 - 5x^3 + 3x^2 - 4x - 6) dx$$

$$113. \int_{-4}^4 |x| dx$$

$$114. \int_{-\pi/2}^{\pi/2} \sin x dx$$

$$115. \int_0^5 e^x dx$$